

TEACHING MATHEMATICS FOR BEAUTY, CREATIVITY AND INVENTIONS

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Abstract

Teaching mathematics without providing for creativity denies all students, especially mathematical-oriented and talented students, the opportunity to appreciate the beauty of mathematics and fails to provide the student an opportunity to fully develop his or her talent. The essence of mathematics is thinking creatively, not simply arriving at the right answer. Mathematicians have always felt a strong creative aspect in their subject, in this vein, encouraging mathematical creativity in addition to computational fluency is essential for students to develop a deep conceptual understanding. This paper examined the teaching of mathematics, and what the stakeholders should do to enhance creativity; it advocated that motivation, engagement, relative freedom, independence of thinking, and flexible thinking be encouraged and developed in the mathematics classroom to foster creativity. It went further to show that Mathematics creativity is displayed by students who think divergently. These are students who generate ideas, conjectures, algorithms, or problems or solutions; consequently, a lot can be done in the mathematics classrooms that can nurture the creativity potential of our students.

Education is the greatest force that can be used to bring about change. It is also the greatest investment that a

nation can make for the quick development of its economic, political, sociological and human resources. It

was in realisation of this that a National Policy on Education was formulated for the country. The policy seeks the inculcation of national consciousness and national unity; the inculcation of the right type of values and attitudes for the survival of the individual and the Nigerian society; the training of the mind in understanding of the world around; and the acquisition of appropriate skills, abilities and competence both mental and physical as equipment for the individual to live in and contribute to the development of his society.

In modern times, education has been marketed in terms of career advancement, higher salary scale, and competitive advantage in the job market. But actually education is about learning, preparing for wise leadership and personal achievement, opening one's mind to new ideas, and strengthening the competitive advantage of a nation. Israel, (2012).

One of the goals of any educational system should be fostering creative persons. Creativity is the ability to think and act in ways that are new and novel. In our minds, there are two kinds of creativity, **innovation and invention**. Innovation is thinking creatively about something that already exists. Innovation is closely associated with and flows out of creativity – (Marilyn Herasymowch & Associate Institute, 2011). In practice, in most instances innovation can result from discovery or invention. Innovation works as a value adding process leading to

commercialisation of ideas and inventions and ultimately to better living conditions.

Creativity is the essential source of all inventions and innovations and is derived from imaginative thought rather than from rational thought. Creativity creates change, Creativity is a process:

- Awareness and interest: the recognition of a problem or situation which engenders curiosity and compelling interest to do something about it
- Preparation and understanding: early analysis, diagnosis and planning which increases understanding of how the task might be approached and the situation remedied
- Absorption and incubation: working out the problem by seeking out possible solutions

Creativity is a dynamic property of the human mind that can be enhanced and should be valued. It can be either strengthened or deteriorated. Luckily, nature of Mathematics provides a suitable platform for developing creativity. (Ginsburg, 1996), rightly puts it, “The essence of Mathematics is thinking creatively, not simply arriving at the right answer”.

Mathematics is a science that deals with numbers and their interrelations, combinations and operations, data, measurements, quantity, structure, space and generalizations. The science is extensively used in the fields of finance,

engineering, industries, robotics, astronomy, business, information technology, and various other fields. Mathematicians analyze data, study patterns and develop mathematical models that not only help to further human knowledge, but are also utilized for solving problems in various fields.

The Merriam-Webster (2014) dictionary defines Mathematics as the science of numbers and their operations, interrelations, combinations, generalizations, and abstractions and of space configurations and their structure, measurement, transformations, and generalizations. Looking at the nature of Mathematics, Mathematics is not an invention; just like “Discoveries and laws of Science” are not considered inventions; inventions are material things and processes - Bells, (2015). When civilization began to trade, a need to count was created. When humans traded goods, they needed a way to count the goods and to calculate the cost of those goods. The very first device for counting numbers was the human hand, counting on fingers. To count beyond ten fingers, mankind used natural markers, rocks or shells. From that point, counting boards and the abacus were invented.

Many students dislike classes in mathematics. They give a wide variety of reasons for this and among the most mentioned ones are that mathematics is hard, mathematics is boring and mostly irrelevant. Part of this problem stems from misconceptions about mathematics. It is described as inflexible

and formulaic as opposed to fun and creative. As teachers of mathematics it is our duties to counteract those prejudices and create a fertile learning environment, the writers continually seek to inspire students and convince them that mathematics in all its forms is worthwhile.

Mathematics expresses itself everywhere, in almost every facet of life - in nature all around us, and in the technologies in our hands. Mathematics is the language of science and engineering - describing our understanding of all that we observe.

Mathematics is important as a discipline, as a field full of wonder and beauty, as a tool for modeling our world, as a prerequisite for knowledgeable citizenship in a participatory democracy, and as a means to better jobs and a better quality of life. We hold strong views on the importance of education in general and mathematics education in particular. Creativity is a dynamic property of the human mind that can be enhanced and should be valued. It can be either strengthened or deteriorated. Nature of mathematics provides a suitable platform for developing creativity.

Mathematics is a particular way of thinking and all children everywhere do it quite naturally. From their earliest encounters, children explore the abstractions of mathematics. Parallel to the development of language skills is the development of concepts related to basic areas of mathematics.

An important mathematical concept that infants develop is pattern.

Pattern is the underlying theme of all mathematics and science. It is our ability to discover and recognize patterns that help us understand how our world works in logical and predictable ways. Experiences with observing and making sense of patterns are what helps young children become logical thinkers who can reason and think critically.

Often called the language of the universe, mathematics is fundamental to our understanding of the world and, as such, is vitally important in a modern society such as ours. Everywhere you look, it is likely mathematics has made an impact, from the faucet in your kitchen to the satellite that beams your television programs to your home. As such, great mathematicians are undoubtedly going to rise above the rest and have their name embedded within history.

Mathematics by most people is viewed as a rigid, formulaic subject without any bearing on real life. While it is correct that part of mathematics consists of rules, logical structures and formulas, most of mathematics center around the ability to develop tools that are applicable to a wide variety of problems – (Brunkalla, 2009). Thus mathematics includes the ability to abstract real world situations, choose the proper mathematical tool for the solution and to interpret abstract results in the light of reality. Most of these abilities are included in Froebel's considerations and teachings.

Before scientists can develop medicines or engineers can advance

technology, they throw numbers onto whiteboards using concepts laid out by mathematicians sometimes centuries earlier. Generations of school children will disagree, but no other field of study has played a bigger role in changing the course of history as mathematics.

Brunkalla, (2009) observed that the awareness of the importance of creativity in mathematics grew noticeably, to the extent that 91% of the students who participated in a survey agreed that creativity is part of mathematics.

The Beauty of Mathematics Patterns

Some people say mathematics *is the science of patterns*; this is a pretty accurate description. Not only do patterns take on many forms, but they occur in every part of mathematics. But again patterns occur in other disciplines as well. They can be sequential, spatial, temporal, and even linguistic. Mathematics seeks to discover and explain abstract patterns or regularities of all kinds. Mann, (2006) opined that when students begin to explore the structure of Mathematics, they begin to explore the beauty of the domain and develop a sense of mathematics.

In nature 1, 1, 2, 3, 5, 8, 13, 21, 34, 55,89,... This is one of the most famous patterns in Mathematics: the *Fibonacci sequence*. It was spotted by Leonardo Pisano, now better known as Fibonacci – Thomas, (2014). In this sequence, each number is derived from adding the previous two numbers. This sequence of numbers can be found in

many natural patterns like in pineapples, sunflowers, nautilus and pine cones. Our eyes are usually drawn to objects that are symmetrical. Moreover, recognizing number patterns is an important problem-solving skill. If you recognize a pattern when looking systematically at specific examples, that pattern can then be used to make things easier when needing a solution to a problem.

Mathematics is especially useful when it helps you to predict or make educated guesses, thus we are able to make many common assumptions based on reoccurring patterns. For example, the first pattern below present a form of beauty associated with Mathematics.

$$\begin{aligned} 12,345,679 \times 9 &= 111,111,111 \\ 12,345,679 \times 18 &= 222,222,222 \\ 12,345,679 \times 27 &= 333,333,333 \\ 12,345,679 \times 36 &= 444,444,444 \\ 12,345,679 \times 45 &= 555,555,555 \\ 12,345,679 \times 54 &= 666,666,666 \\ 12,345,679 \times 63 &= 777,777,777 \\ 12,345,679 \times 72 &= 888,888,888 \\ 12,345,679 \times 81 &= 999,999,999 \end{aligned}$$

Source: Michelle Tsivgadellis (gofigurewithscipi.blogspot.com)

That's an easy pattern to see! However, using a calculator to find out what would happen if we multiplied 12,345,679 by 90, by 99 or by 108; it was discovered that another pattern develops.

Here is a similar pattern that uses the multiples of 9. Looking at the first digit of each answer; it increases by 1 each time. Observing the last digit of each answer, another pattern appeared.

$$\begin{aligned} 987,654,321 \times 9 &= 08\ 888\ 888\ 889 \\ 987,654,321 \times 18 &= 17\ 777\ 777\ 778 \\ 987,654,321 \times 27 &= 26\ 666\ 666\ 667 \\ 987,654,321 \times 36 &= 35\ 555\ 555\ 556 \\ 987,654,321 \times 45 &= 44\ 444\ 444\ 445 \\ 987,654,321 \times 54 &= 53\ 333\ 333\ 334 \\ 987,654,321 \times 63 &= 62\ 222\ 222\ 223 \\ 987,654,321 \times 72 &= 71\ 111\ 111\ 112 \\ 987,654,321 \times 81 &= 80\ 000\ 000\ 001 \end{aligned}$$

Source: Michelle Tsivgadellis (gofigurewithscipi.blogspot.com)

Recognizing, deciphering and understanding patterns are essential for several reasons. First, it aids in the development of problem solving skills. Secondly, patterns provide a clear understanding of mathematical relationships. Next, the knowledge of patterns is very helpful when transferred into other fields of study such as science or predicting the weather. Without being able to recognize the development of patterns, the ability to be proficient in Mathematics (generally) will be limited. Since all Mathematics are based on patterns, some mathematical problems could be used to investigate developing patterns.

In the next example, we begin by multiplying one by one; then 11 by 11, and so forth. Each time we multiplied, the number of digits in the multiplier and the multiplicand increases and a beautiful pattern progresses in the answer (product). This multiplication pattern forms a triangle.

$$\begin{array}{r} 1 \times 1 = 1 \\ 11 \times 11 = 121 \\ 111 \times 111 = 12321 \\ 1111 \times 1111 = 1234321 \\ 11111 \times 11111 = 123454321 \\ 111111 \times 111111 = 12345654321 \\ 1111111 \times 1111111 = 1234567654321 \\ 11111111 \times 11111111 = 123456787654321 \\ 111111111 \times 111111111 = 12345678987654321 \end{array}$$

Source: Michelle Tsivgadellis (gofigurewithscipi.blogspot.com)

Another interesting pattern is given below. In this one, instead of multiplying by 1, then 11, then 111, the answer (product) looks like the multiplier in the pattern above.

$$\begin{array}{r} 1 \times 9 + 2 = 11 \\ 12 \times 9 + 3 = 111 \\ 123 \times 9 + 4 = 1111 \\ 1234 \times 9 + 5 = 11111 \\ 12345 \times 9 + 6 = 111111 \\ 123456 \times 9 + 7 = 1111111 \\ 1234567 \times 9 + 8 = 11111111 \\ 12345678 \times 9 + 9 = 111111111 \\ 123456789 \times 9 + 10 = 1111111111 \end{array}$$

Source: Michelle Tsivgadellis (gofigurewithscipi.blogspot.com)

Yes, we are multiplying by 9 each time, but look at the number being added, and count the number of ones "1s" you see in each answer. This is the beauty in Mathematics, it is ordered, methodical and precise.

Competencies Required of Creative Mathematicians:

Kilham, (2015) outlined traits that all great inventors have in common, they are:

✚ Unleash your curiosity, quest for knowledge and propensity for noticing things. Einstein wrote: "The important thing is not to stop questioning." You should also notice things, however unrelated to your quest they may seem.

✚ Project your mind into imagination space, focusing on all the interrelated aspects. To be a creative mathematician, you must forcefully move your mind beyond existing thinking about the subject.

✚ Focus on the practical, useful, needed and beautiful. Very often inventions and other creations start out answering to a major need or a broad interest: this is Mathematics in action – Looking for solution to problems.

✚ Be persistent. Don't give up as Mathematical proof is fundamentally a matter of rigor.

Often real situations can be quite complex and can be explored and made sense of using relatively straight forward Mathematics. Wake, (2005) sighting Niss, elaborated on competencies required of a creative mathematician:

(i) **Thinking and Reasoning:** This involves not only asking questions that are characteristic of Mathematics but when it is appropriate to ask such questions and the kinds of answers that one might expect.

(ii) **Argumentation:** This involves following and understanding mathematical arguments and having an understanding of the nature of proof.

(iii) **Communication:** This involves expressing one's own Mathematics in a variety of forms and making sense of the Mathematics of others.

(iv) **Modelling:** This includes involvement in all aspects of the modelling process as the conceptualisation of mathematical modelling and how it relates to applications of mathematics can effectively improve the teaching and learning of mathematics at all levels.

(v) **Problem solving and Posing:** Posing, formulating, defining mathematical problems of different types in different domains is at the core of this competency.

(vi) **Representation:** This involves being able to work with a wide range of mathematical representations, both interpreting those of others and being able to develop appropriate representations to communicate mathematical ideas to others.

(vii) **Using Symbolic, Formal and Technical Language and Operations:** This involves working with a range of mathematically technical language to develop and communicate mathematical arguments.

Research has also shown that creative people do not like to work in a conventional way. They have a desire to shake things up. They are dissatisfied with the status quo. They are restless, rebellious, courageous, diligent, and independent (Cangelosi, 1996; Meissner, 2000; Yushau, Miji & Wessels, 2003).

These competencies appear to be potentially useful in identifying the ways in which creative students are supposed to be working mathematically.

Injecting Creativity into Mathematics Lessons

A lot has changed in Mathematics lessons over the last generation, instead of learning procedures by rote, children are being taught to understand the methods they are using, and that is surely a good thing; yet there is still something missing. It seems that very few children are being taught to love Mathematics or to get excited about it. Eastaway, (2012) opined that

“There's a widespread perception that Mathematics is simply a set of methods that must be learned in order to be able to cope with life after school.” Why is this? In many cases it is because the teachers themselves have little love for the subject.

If Mathematics is to be a creative subject then we have to regard it as a subject where it is OK to get things WRONG. If students never make mistakes, they are not discovering anything new. The history of civilisation shows that we only create things when we are allowed to make mistakes. Indeed many of the greatest inventions and discoveries came as a direct result of things going wrong.

Thinking mathematically is considered (by many people) to be critical for everyday life skills. People use mathematics skills daily to identify problems, look for information that will help solve problems, consider a variety of solutions, and communicate the best solution to others.

However, the connection between the mathematics learned at school and the mathematics used in daily life is more often than not missing. To bridge this gap, mathematics classrooms should provide practical experience in mathematical skills that are a bridge to the real world. Also they should allow explorations that develop an appreciation of the beauty and value of mathematics (Beal, 1998; Yashua, et al 2003)

Mathematics teachers need to stimulate children's curiosity. Students should experiment, and find out by themselves that sometimes things go wrong, and asking "why?". Moreso, students should be motivated to think creatively, and be given relative freedom to chart their ways in Mathematics class. Recent educational and psychological research highlights the role of multiple affective variables and specifically of motivation towards learning in pursuing educational goals (Backacrts, 2001). Motivation refers to the forces encouraging a person for which a student works persistently to reach a desirable result (Wolters and Rosenthal, 2000).

Emphasis should therefore be placed on creating authentic learning situations where students are thinking, feeling, and doing what practicing professionals do (Renzulli, Leppien & Hays, 2002). The fundamental nature of such authentic high-end learning creates an environment in which students apply relevant knowledge and skills to the solving of real problems (Renzulli, Leppien, & Reis, 2004). Solutions to the real problems also entail problem-finding, as well as problem-solving. These, surely, are the secrets to making Mathematics more and more an exciting and creative subject at school.

Conclusions

Patterns and structures are fundamental to Mathematics. They allow mathematicians to spot when something interesting is going on, to

identify the core of a problem and to generalise from a specific example to a more general understanding.

Often spotting a pattern in a problem is the first step to understanding the underlying structure involved. And here lies the strength of Mathematics: the same mathematical structures can appear in wildly different settings. People love to spot patterns; it is something that we are intuitively good at. For this reason, Mathematics teachers should encourage, and allow flexible, independent thinking from their students in their approach to problem-solving.

To this end, Mathematics teachers should cultivate the habit of motivating and arousing the interest of students both during and after the Mathematics class. They should zealously and enthusiastically communicate the importance of Mathematics to students in real life situation; in such a way that future students would love and grow to be mathematicians - an expert in the field of mathematics; a person who is highly skilled and knowledgeable.

Recommendations

The following are recommended to improve and foster creativity in Mathematics class:

- ✚ A zealous or enthusiastic teacher should communicate the importance of mathematics to students in real life situation.
- ✚ Mathematics teachers should cultivate the habit of motivating and arousing the interest of

students during the teaching of mathematics in the classroom.

- ✚ Teachers should expose students' experiences which are pleasant and give satisfactions, thus, students will be motivated to learn in an atmosphere of freedom.

- ✚ Students should be allowed extra chances to complete a task successfully (make the effectiveness of their thinking and not the speed of their thinking the valued trait).

- ✚ Don't average disparate skills together. Rather than trying to weight creativity versus rigor, provide distinct feedback on each creative skill (e.g., problem posing, conjectures, reasoned arguments, clear notation, use of definitions, cycling through increasingly complex observations and proofs, persistence and initiative)

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