TEACHING BASIC MATHEMATICS FOR SOLID FOUNDATION AND CREATIVE THINKING

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Abstract
Limited reasoning is a big hindrance to Educational development and a major challenge, the Nigerian Educational system needs to overcome to ensure solid foundation in Mathematics and encourage creative thinking. Using examples from Division, Expansion, brackets and Equations, the paper shows how limited reasoning can be a hindrance to Teachers' effectiveness in teaching students attempting creativity.

Creativity is vital to sustenance of life, giving limited recourses and many competitors using the recourses. Creativity is very important for the sustenance of entrepreneurship and the development of Nations. Making a habit of limited thinking is detrimental to utilization of the brain for creative endeavours. The paper shows how limited thinking hinders teachers' effectiveness in teaching students attempting creativity.

A Review of Division – The Teacher and His Pupils

Division of numbers is an important foundational topic in the study of mathematics. Consider the division of 216 by 3.

i.e. $\overset{H}{2}\,\overset{T}{1}\,\overset{U}{6} \div 3 = \frac{216}{3}$

The 2 hundred (H) is too small to go round 3 persons each having at least a hundred. Therefore, change the two hundreds to next smaller unit i.e. Tens (T). Since 10 tens equals one hundred, 2 hundreds = 2 x 10 tens = 20 tens. The 20 tens together with the (1) ten on ground will make a total of 20 + 1 = 21 tens in the number 216.

3 into 21 tens is (7) tens. Next 3 into 6 units is (2) units and the division is completed.

$$\therefore \frac{216}{3} = 72$$

A teacher statement of brevity for the division of 216 by 3 is start from left hand side, 3 into 2 is too small, take the (2) and the next digit together i.e. 21 (twenty one), 3 into 21 is (7), 3 into 6 is (2).

$$\therefore \frac{216}{3} = 72$$

The working may be set out in tabular form as follows:

```
     72
   ______
3 | 216
  | 21    (-)
  |  6
  |  6   (-)
  | -  -
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In this tabular form, a teacher says 3 into 2 (too small), take the next digit with it i.e. 21. 3 into 21 is (7), write the (7) on top. To get the remaining number when 21 is divided by 3, he multiplies the divisor (3) and the quotient (7) i.e. 3 x 7 = 21 and arrange the 21 in the appropriate place, in this case, the first two places from left, division has reached. He then subtracts the 21 obtained from 3 x 7 from the number in the places division has reached, in this case the first two digits from the left of the number $\substack{216\ (I.e.21)\\ 21-21=0}$ $(no\ remainder)$.

To continue the division, the teacher brings down the next digit (6) and say, 3 into 6 is (2) and writes the (2) on top in line with the 6. Next, subtract 6 from 6 gives zero.

$$\therefore \frac{216}{3} = 72$$

Some Problems of Un-clarified Brevity in Division

A basic requirement for creativity is ability to connect previous learning with new situations in coherent forms. For the division of 216 by 3, if the teacher says 3 into 2 (cannot), a student attempting creativity may be thinking why cannot? He may recall similar maths question that says divide 2 by 5 and the answer was 0.4. The matter is further worsened when the teacher say "add the next digit" when he wants to say take the digit and the next one together. Then in the $\frac{216}{3}$ example, the student expects the result to be 2 + 1 = 3 but the teacher says 21! He puts the (7) on top. Next, the teacher says 3 x 7 = 21 in the tabular form, the student is thinking why did the teacher multiply 3 by 7? The student may not ask questions and some teachers may not clarify their purpose for some steps in the working process but expects students to follow the examples. Next, the teacher arranges the result 3 x 7 = 21 under 216 as follows:

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    7
   ______
3 | 216
  | 21
  |
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And say "subtract" gives 6. The student is thinking !, he recalls from previous studies that 216 – 21 should be arranged as $\begin{array}{r} 216 \\ \underline{21-} \\ 195 \end{array}$ but the teacher say in his tabular division, $\begin{array}{r} 216 \\ \underline{21\ -} \\ 6 \end{array}$ is 6. Yet, the student did not ask question to enable the teacher explain the concept more clearly. Recall that in the division of 216 by 3, when the first digit (2) was too small, the teacher did not put down zero (0) before taking the next digit with it. He said 3 into 2 (too small), 3 into 21 (7). Next, consider the teacher's second example, divide 615 by 3 i.e. $\frac{615}{3}$, the teacher says 3 into 6 is (2), 3 into 1 is too small, put down zero (0), take the (1) and the next digit (5) together i.e. 15, 3 into 15 is (5).∴ $\frac{615}{3} = 205$

Here again, the student expectation of (25) is disappointed. Because in the first example, in dividing 2 by 3 since 2 was smaller than 3, the teacher said cannot and did not put down zero (0), following the first example why should the teacher now put zero (0) when 3 "cannot" go into 1 to get result 205?

Students attempting creativity are often very anxious to work out the result of a problem they recognize as similar to a previous one and when their attempts are met with frequent disappointments from the teacher, it may cause internal frustration, loss of confidence and a conclusion that the mathematics process is not trustworthy. Hence they may lose interest in mathematics. In this regard we buttress our statement by a quotation from Souper (1976). "In most test, there is normally one right answer. The divergent thinker, the one who ask awkward questions, whose mind is apt to go off at a tangent and to think creatively, is at a disadvantage."

Un-clarified Assumptions in Previous Studies Resurfacing as a Barrier to Current or Future Learning

Misconceptions in expansion

To teach expansion, following examples of some teachers, a teacher says

a × (b + c) = a x b + a x c

a × (b – c) = a x b – a x c

Through practice and examples of the teacher, the student learns that a multiplying number outside a bracket spreads in multiplication over the numbers in the bracket. The teacher assumes that seeing the examples, the student should understand what is to be learnt and the students may not ask questions on areas of doubt.

Following these examples, the student may carry on and succeed for many years within boundaries of teacher's example, but when the students encounter new situations, the problem of unasked questions or un-clarified questions will arise. For example, in our teaching experience, a National Diploma 1 (ND 1) student in one of our classes in 2017 queried a teacher's factoring of $4 \times 2^n \times 2^1 - 2^n \, as \, 2^n(4 \times 2^1 - 1)$ in an indices class, after careful observation, the teacher asked the student to express her difficulties more clearly, the student said the reason she did not accept the factoring even after a lot of explanations was that drawing from her previous knowledge, accepting the factorization will contradict her expectation of the expansion which she expected to be:

$2^n(4 \times 2^1 - 1) = 2^n \times 4 \times 2^n \times 2^1 - 2^n \times 1$ Ekhoragbon (2017). What the student did not know or was not told in her secondary education was that multiplication does not distribute over numbers in multiplication and division.

Misconceptions Arising from Reversibility Equivalence/Transfer in the Solutions of Some Equations

Consider the equation $x - 7 = 3x - 5$ - (1)

The equation is a linear equation in one variable x. To solve the equation, we need to separate terms with variables from terms having numbers only. Although putting the variable on any side will not change the solution, it is common practice to separate the variables on the side that when simplified, its coefficient will be positive, this is done to avoid dividing by a negative number.

To solve the equation (1), if we decide to put all x on the right, the x on the left will move to the right and -5 on the right will move to left, the other terms are already in appropriate places, so need no transfer.

To achieve this, we recall that if two things are equal and equal things are added to both sides, they will remain equal. Therefore, to transfer x on the left to right, add $-x$ to both sides. To transfer -5 from right to left add +5 to both sides of the equation.

$i.e\ x - x - 7 + 5 = 3x - x - 5 + 5$

$x - x = 0 \quad , \quad -5 + 5 = 0$

The equation becomes

$-7 + 5 = 3x - x$ - (2)

Comparing equations (1) and (2), we see that x on the left of equation (1) has moved to the right of (2) as $-x$ and -5 on the left of equation (1) has moved to the right of (2) as +5.

Therefore if a term of an equation moves from one side of equality to another, the sign changes. For convenience in the solution of equations, mathematics teachers often use reversibility equivalence of equality to move variables to the left when they are on the right. For example, if an equation is solved to the point 12 = -3 x, the teacher will then write -3 x = 12.

To the teacher, it is very clear that if a = b then b = a. He then proceeds to write.

$$x = \frac{12}{3}$$

$$x = -4 \quad Answer$$

But what is very clear to the teacher may not be clear to the student. In our teaching experience, and ND 1 student in one of our classes asks this question as a general question bothering her in a Maths class on combination.

“Sir, why is it that if we solve an equation and obtain -3= x they will now say x = -3 is the answer but they said if a term moves from one side of an equation to another, the sign changes but why did the signs not change in this case?” Ekhoragbon (2017).

This is a question many Nigerian students we have taught have not asked, even we as teachers never thought of it that way until our attention was drawn to it by a student in 2017! Note that her question was unrelated to the topic and examples for that day.

This shows that Nigeria have some intelligent students who suppress questions if they do not have a conducive class environment to express themselves freely and are confused when their questions are not effectively tackled. To resolve this apparent contradiction, the teacher may proceed as follows. Explain to the student that to solve for x means to separate x alone on its side such that it is positive and has a coefficient of +1. Therefore, -3 = x is solved. So we can use the reversibility equivalence of equality i.e. if a = b then b = a to say that x = -3 is the solution.

If we use transfer to move x to the left and -3 to the right, we have

$$-x = +3$$

$$x = \frac{+3}{-1}$$

$$x = -3 \quad same\ result$$

If the equation is not solved and the variable (x) is on the right. For example 12 = -3x, we can use transfer to bring it to the left.

$$i.e\ 3x = -12$$

$$\therefore x = \frac{-12}{3}$$

$$x = -4$$

Or if we use reversibility, we have

$$-3x = 12$$

$$x = \frac{12}{-3}$$

$$x = -4 \quad Same\ result.$$

Thus if a solution of an equation has (x) negative on the right, use transfer of terms to make it positive on the left and get the solution. If a solution of an equation has (x) positive on the right, use reversibility of equality to get the solution.

Misconceptions in Using Bracket

In teaching brackets, a common practice is to say:

If a bracket is removed after a plus sign, the sign of the term inside the bracket does not change e.g. remove the bracket and simplify $2a + (6a - 5b + 4)$

Solution:

$$2a + 6a - 5b + 4$$

$$Answer \quad 8a - 5b + 4$$

If a bracket is removed after a minus sign, the sign of all terms in the bracket changes when removed e.g. remove the bracket and simplify.

$$2a - (6a - 5b + 4)$$

Solution:

$$2a - 6a + 5b - 4$$

$$Answer \ - 4a + 5b - 4$$

Some teachers may talk briefly about related rules for insertion of brackets. In all these, hardly do they teach how to know when to use a bracket.

It is true that to add 7a and 2a – 4, students should write:

$$7a + 2a - 4$$

It will be incorrect of the student who is asked to subtract $2a - 4$ from $7a$, to write:

$7a - 2a - 4$ because the subtraction will then affect $2a$ only whereas we ought to subtract $(2a - 4)$. Thus the correct response is $7a - (2a - 4)$. To raise $2b$ to power 5, some students write $2b^5$ when they ought to write $(2b)^5$. Since the students know that the square of a is written as a^2, when asked to square $\frac{2}{3}$ some write $\frac{2^2}{3}$ when they ought to write $(\frac{2}{3})^2$. All these indicate that some students still have difficulties using brackets effectively.

1. A bracket is needed when we subtract an expression that has two or more terms
2. A bracket is needed when we want to raise a product of two or more terms to a power
3. A bracket is needed when we want to raise the division of two terms to a power e.g. to divide a by b and raise to power 3, we should write $(a \div b)^3$ not $a \div b^3$.
4. To raise a fraction to a power, a bracket is needed

Implementing Broad Thinking in Teaching Maths

If a problem has more than one solution, the best method is treated in detail, the others should be illuminated and given as assignment for verification.

Examples should be carefully selected such that when solved will teach multiple creative skills in mathematics. The teacher needs to do research on problem construction or selection to obtain a small set of basis examples whose solution will give a comprehensive view of skills required so that mathematics be not presented as "infinite number of fragmented parts without summarizing relationship and the problem solving process in magical ways." Dosomah 2008.

The purpose for each step in the working process should be clarified and Teachers should use their experience to clarify possible misconceptions in the teaching process.

Observations

5. Nigerian students do not ask questions because majority of them do not think deeply on what is presented to them
6. Some students having difficulties relating new maths lessons to previous ones may suppress questions
7. Teachers seem to settle for right solution method and may not think about possibilities of misconception arising from their presentation when students do not ask questions
8. In a situation where many students do not ask questions few that are having difficulties relating lessons may not be encouraged to ask so as not to be branded "slow to understand", "a class disturbance" or "too know."
9. The teacher's attitude in managing questions is critical to the success of a conducive class where students are free to ask questions and learn effectively.

Conclusion

Limited reasoning of teachers and students is a big hindrance the Nigerian Educational System must overcome for sustenance of a progressive educational system, encourage creativity and entrepreneurship.

Recommendations

In light of the issues discussed in this paper, the following recommendations are made towards solid foundation and creative thinking in teaching Basic Mathematics.

1. Students should be encouraged to think creatively.
2. Students should be brought up in intelligent ways such that they are quick to identify relationships, fast and accurate, good in analysis and can maintain focus in multi-tasking. Such approach will be a good foundation for creativity and success in entrepreneurship.
3. Teachers should look beyond correct answers to examine perspectives of students in their responses so that they can understand their learning difficulties and correct misconceptions.
4. Teachers should help by using their experience to simplify learning, ensure smooth transition from one concept to another, ask leading questions on possible misconceptions, clarify them and maintain a conducive class environment.

References

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